The most probable value of z is that one which will make P a maximum. P is a maximum when $(\varepsilon_1^2+\varepsilon_2^2+\ldots\ldots\varepsilon_n^2)$ is a minimum; or, by equating to zero, the first derivative, when $\varepsilon_1+\varepsilon_2+\ldots\ldots\varepsilon_n=0$. That is when $z-w_1+z-w_2+\ldots\ldots z-w_n=0$. Solving for z we find

$$z=(w_1+w_2+\ldots\ldots w_n)/n. \tag{17}$$

Hence it appears that in the long run the best way to solve the markings of judges of contests is simply to take the arithmetical means of the markings when the merits of the contestants are compared with ideal standards.

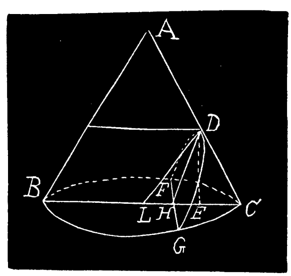
The methods for successive approximations have already been discussed in this journal.

The University of Chicago.

A METHOD OF DEFINING THE ELLIPSE, HYPERBOLA AND PARABOLA AS CONIC SECTIONS.

By W. W. LANDIS, A. M., Professor of Mathematics in Dickinson College, Carlisle, Pa.

ABC is a right circular cone, the angle at the vertex being 2α. DFG is a plane section making an angle θ with the axis of the cone. Take D as the origin, DH as the axis of x, and a perpendicular through D as the axis of y. We seek to find a relation between x and y, the parameters being $g(=AD)$ and α, and the variable parameter θ. In the circle BGC, $y^2=(a\pm b)c=ac\pm bc$......(1), where $a=BL$, $b=LH$, and $c=HC$. In the isosceles triangle DLC, $x^2=d^2\mp bc$ ......(2), where $d=DC=DL$. Adding (1) and (2), $x^2+y^2=ac+d^2$......(3).



Now $c=x\sin\theta+d\sin\alpha$,

$d=x\cos\theta\sec\alpha=x\cos\theta/\cos\alpha$, and

$a=2g\sin\alpha$.

Making this substitution we get

$$x^2+y^2=x^2\frac{\cos^2\theta}{\cos^2\alpha}+2g\sin\alpha\left[x\sin\theta+x\frac{\cos\theta\sin\alpha}{\cos\alpha}\right]$$

$$\text{or } x^2\left[1-\frac{\cos^2\theta}{\cos^2\alpha}\right]+y^2-2gx\sin\alpha[\sin\theta+\cos\theta\tan\alpha]=0\ldots\ldots (4)$$

which we may write

$$x^2\left[\frac{\sin^2\theta-\sin^2\alpha}{1-\sin^2\alpha}\right]+y^2-2gx\sin\alpha[\sin\theta+\cos\theta\tan\alpha]=0\ \ldots\ldots (5).$$

Now if $\theta=\alpha$, the plane then being parallel to one and only one element, (5) reduces to $y^2-4gx\sin^2\alpha=0$, a parabola of latus rectum$=4g\sin^2\alpha$.

If $\theta>\alpha$ the section cuts all elements and the coefficients of x^2 and y^2 are both positive, and we have an ellipse whose center, axes, and eccentricity are readily found; and in particular if $\theta=90°$, the section is parallel to the base, the coefficients of x^2 and y^2 are unity and we have a circle, whose center is $(g\sin\alpha,\ 0)$. If $\theta<\alpha$, the section cuts both nappes, the coefficients of x^2 and y^2 are of unlike sign and we have a hyperbola.

If $g=0$, (5) becomes $y=\pm x\sqrt{\dfrac{\sin^2\alpha-\sin^2\theta}{\cos^2\alpha}}$....(6). Now if $\theta=\alpha$, (6) becomes $y=0$, a straight line, the limit of the parabola. If $\theta<\alpha$, (6) represents two real straight lines, the limiting case of the hyperbola. And if $\theta>\alpha$, (6) represents two imaginary lines intersecting in the real point (0, 0), which is the limiting form of the ellipse.

The equations (5) and (6) show the dependence of the nature of the conic sections upon the angle which the cutting plane makes with the axis, and the dependence of their shape upon the angle of the cone and the distance from the vertex to the first element cut.

REMARK 1. If the section be a parabola, the foot of the perpendicular from the middle point of the line through D parallel to BC, upon DH, is the focus.

REMARK 2. The eccentricity of any conic section is $\varepsilon=[\cos\theta/\cos\alpha]$.

NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By BENJ. F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and JAMES A. CALDERHEAD, B. Sc., Curry University, Pittsburg, Pennsylvania.

[Continued from November Number.]

LXX. Fig. 31.

$AEM=ACB$ of $AEHC$.

$MOL=ACB=ADK+DKBC=BHI+DKBC$. $LOI=BEK$.

$\therefore ABLM \asymp BCDF+AEHC$.

LXXI. Fig. 31.

$AMPQ \asymp AMOC \asymp AEHC$. $BLPQ \asymp BLOC \asymp BCDF$.

$\therefore ABLM \asymp BCDF+AEHC$.

LXXII. Fig. 31.

$MTE=BSF$. $\therefore BS=MT$. $\therefore AMLB \asymp 2AMTS$.

But $AMTS \asymp EAC+FBC$; since $MTE=BSF$, and $AEM=ACB$.

$\therefore 2AMTS \asymp 2EAC+2FBC \asymp AEHC+BCDF$.

$\therefore AMLB \asymp BCDF+AEHC$.